CHAPTER 1 (x÷=−+) (x÷=−+)

Overview of Solve It! Instruction

National assessments show that students in the United States generally perform poorly in mathematics. Results also show that students perform particularly poorly on assessments of mathematical problem-solving. This is not a new phenomenon.

The poor mathematics performance of students in U. S. schools has been demonstrated consistently on state, national (e.g., National Assessment of Educational Progress; NAEP, 2000, 2003, 2007), and international mathematics tests (e.g., National Center for Education Statistics; NCES, 2003, 2007). Students in urban schools, particularly students with disabilities, are usually the lowest performing, putting them at even greater risk for poor academic outcomes and school dropout.

There remains a persistent performance gap in mathematics between poor students from diverse cultural backgrounds and their White, non-Hispanic middle-class counterparts and students with and without disabilities (NAEP, 2003, 2007). For example, NAEP 2003 results indicated that White non-Hispanic students scored 40 points higher, on average, than African American students. On the NAEP 2007, among eighth graders, 42% of White non-Hispanic students scored proficient or better, compared to 11% of African American, 15% of Hispanic, and 16% of Native American students. On the NAEP 2003, 71% of students with disabilities, contrasted with 27% of students without disabilities, scored below the basic level. NAEP 2007 indicated that 40% of participating Grade 4 students with disabilities scored below the basic level compared to 15% of Grade 4 students without disabilities. The gap widened in Grade 8 and again in Grade 12, with 83% of students with disabilities scoring below the basic level compared to 36% of their nondisabled peers.

In keeping with the No Child Left Behind Act (NCLB, 2001), all students, including students with disabilities, must meet high standards in mathematics as measured by state-administered achievement tests. To meet these standards in mathematics, at risk students and students with disabilities in urban schools—who vary considerably in ability, achievement, and motivation—must develop the necessary problem-solving skills needed not only to perform well on mathematics assessments but also to apply these skills successfully in real world settings.

To address students' mathematical needs, the National Council of Teachers of Mathematics (NCTM) issued several national reports that focused on a fundamental shift in mathematics instruction from basic skills to mathematics comprehension and application. NCTM established national standards and goals for assessment and instruction that emphasized conceptual development in mathematics, communication about mathematics, and mathematical problem solving (NCTM, 1989, 1991, 2000). NCTM called attention to the dismal mathematics performance of students and recommended a more meaningful cognitive approach to teaching and learning mathematics. While these standards have helped educators to improve students' mathematics learning in many ways, one area—mathematical problem solving—remains a major concern that poses significant challenges.

The NCTM Standards have had a far-reaching effect on mathematics curriculum, instruction, and assessment. In the classroom, researchers have noted that changes have occurred specifically in the content of mathematics instruction, teachers' pedagogical perspective, the learning experiences provided to students, the time allocation for instruction, and assessments of student performance (U.S. Department of Education, 2008). Specifically, the NCTM Standards and other mathematical reform efforts have resulted in an emphasis on:

- Teaching mathematical concepts and problem solving.
- Using facilitated and guided learning experiences rather than didactic instruction.
- Encouraging active involvement in the learning process.
- Measuring student progress in authentic ways.

Solve It! is a problem-solving instructional program that helps teachers make these changes and respond meaningfully to students' mathematical needs. Students may have many needs related to mathematical problem solving, such as:

- **Poor reading comprehension.** Students may need to be taught various reading strategies that will help them understand the problem (e.g., slowing their reading rate, rereading difficult parts of the problem, focusing on important information by underlining or taking notes).
- **Gaps in mathematical knowledge and skills.** Students may not have the mathematics vocabulary and may need instruction in various terms used in mathematics problems. They also may need to be taught how to recognize that they do not understand the relationships among mathematical terms and quantitative concepts expressed in a problem. They should know what questions to ask and how to ask these questions if they do not understand the problem.

Solve It! addresses these and other needs by directly teaching students cognitive processes and metacognitive strategies that are essential for solving mathematical problems. The framework for **Solve It!** is derived from cognitive theory and reflects the notion that effective and efficient mathematical problem solving depends on the ability to select and apply task-appropriate cognitive processes and metacognitive strategies for understanding, representing, and solving problems (Brown, 1978; Flavell, Miller, & Miller, 1993; Mayer, 1985). Both cognitive processes and metacognitive strategies are related to successful mathematical problem solving (Krawec, 2010; Montague & Applegate, 1993a, 1993b; Slife, Weiss, & Bell, 1985; Sweeney, Krawec, & Montague, 2011; van Garderen & Montague, 2003).

Cognitive Processes

Cognitive processes are integral to the development of declarative and procedural knowledge of arithmetic and the ability to apply this knowledge to word problems. Math problem solving is composed of two problem-solving phases: problem representation and problem execution (Mayer, 1985).

Problem Representation

A primary difference between good and poor problem solvers is the ability to represent problems (Montague, 1997). Problem representation can be defined as the manipulation of the information in the problem and development of a schematic to illustrate the problem using physical objects, a drawing or diagram, and/or mental imaging.

Problem representation processes and strategies are needed to comprehend and integrate problem information, maintain mental images of the problem in working memory, and develop a viable solution path, often by finding alternative and unusual approaches to the problem (Silver, 1985). As such, problem representation involves translating and transforming linguistic and numerical information into verbal, graphic, symbolic, and quantitative representations such as pictures, charts, equations, and operations (Krawec, 2010; Mayer, 1985). Problem representation requires the problem solver to translate linguistic and numerical information into a coherent, integrated problem structure or description. The problem solver must then use this verbal, graphic, symbolic, and/or quantitative representation to generate appropriate mathematical equations and operations (Heller & Hungate, 1985; Mayer, 1985; Montague, Enders, & Dietz, 2014).

Problem Execution

Strategic planning is critical to effective problem solving. After the problem solver accepts the solution plan, problem execution strategies are implemented. Problem execution requires the problem solver to work forward and backward without resorting to trial-and-error or means-ends approaches to problem solving. Metacognition plays a central role in problem execution.

The **Solve It!** approach places particular emphasis on teaching students how to represent mathematical problems by paraphrasing problems, using visualization strategies such as diagram drawing or mental imaging, and hypothesizing or setting up a plan. **Solve It!** incorporates the following cognitive processes:

- **Reading the problem.** Reading the problem implies the ability to understand each part of the problem with the eventual goal of establishing relationships among the parts. When solving word problems found in typical mathematics textbooks, good problem solvers generally begin by reading the problem word by word and then rereading it (or parts of it) one or more times. They also tend to reread sections of the problem as they solve it. **Solve It!** teaches students how to read mathematical problems.
- **Paraphrasing.** The ability to paraphrase a mathematical problem means being able to translate the linguistic information in the problem by rephrasing or restating the problem. Putting the problem into one's own words without changing the meaning of the problem is the test of good paraphrasing. Students should be taught how to paraphrase parts of problems and then tell the story in a way that conveys the meaning of the problem. **Solve It!** teaches students to put the problem into their own words.
- **Visualizing.** Good problem solvers use visualization to help them process the linguistic and numerical information in a mathematical problem and form internal representations in memory. They do this either by drawing a representation on paper or by making a mental image of the problem. These images can be geometric representations, diagrams, tables, figures, or some other type of graphic or schematic display. Some problem solvers imagine the story and actually may see themselves and others as characters in the story. However, it is not sufficient to simply draw a picture. Developing a visual representation that shows the relationships among the components of

a problem is the goal of instruction. These schematic or relational images are the key to successful problem solving (van Garderen & Montague, 2003). **Solve It!** teaches students how to develop schematic representations, either on paper or mentally, that lead to the problem solution.

- **Hypothesizing about problem solutions.** Good problem solvers develop a solution hypothesis that is directly tied to their comprehension of the problem and their integration of the problem information. The reading and representation processes and strategies assist problem solvers in deciding on a solution path. This includes establishing a goal, looking toward the outcome, and setting up a plan to solve the problem. It entails deciding on the number of operations that are needed to solve the problem, selecting and ordering the operations, and then transforming the information into correct equations and algorithms. In **Solve It!** students decide on the best solution based on their representation of the problem information.

- **Estimating the answer.** Estimation is a key strategy in successful mathematical problem solving (Montague & van Garderen, 2003). Estimation helps students validate the process as well as the product of problem solving. It is important to be able to refer back to the question and the goal that was set, decide what one is looking for, and then accurately predict the outcome. Students need explicit instruction in how to stay focused on the type of outcome (e.g., number of yards rather than feet) and then how to predict the answer by using the information in the problem and their projected solution path. In **Solve It!** students are taught how to round numbers up and down so they can mentally compute the answer in round numbers (e.g., 241 ft. + 884 ft. ÷ 3 feet in a yard becomes 200 + 900 = 1,100 changed to 1,200 ft. ÷ 3 ft. = 400 yd.). They then have a ballpark answer to compare with their actual answer. In the example, the actual answer—375 yards—would be compared to 400 (the estimated answer). The student would then decide if the answer is in the ballpark—that is, not too big or too small—and if it has the correct label.

- **Computing.** Computation involves both declarative and procedural knowledge. Students must be able to recall the correct procedures for working through the addition, subtraction, multiplication, and division algorithms and also recall the necessary math facts for accuracy. Calculators facilitate and expedite computation and should be used whenever possible. Students must be taught how to use calculators to compute accurately. In **Solve It!** computation is part of the overall problem-solving process.

- **Checking.** Checking underscores the importance of teaching problem-solving as a recursive process so that students understand that returning to earlier processes and sometimes working backward are typical of successful problem solving. Checking the accuracy of the computation is important as well, even when calculators are used. In **Solve It!** checking the problem entails verifying both the process and the product. That is, students are taught how to check the mathematical problem-solving process to ensure that they have understood the problem, represented the problem accurately, selected an appropriate solution path, and solved the problem correctly.

Self-Regulation Strategies

Cognitive processes may be described as on-line mental activities that are proactive in nature (the "to do" strategies), whereas the metacognitive or self-regulation strategies require reflectivity and reactivity (the "what I am doing" and "what I have done" strategies). Metacognitive or self-regulation strategies differ from cognitive processes by emphasizing self-awareness of cognitive knowledge, deployment or use of cognitive processes during problem solving, and control of processes for purposes of regulating and monitoring performance. Metacognitive strategies are often associated with self-awareness, self-evaluation, and self-regulation (Berardi-Coletta, Dominowski, Buyer, & Rellinger, 1995). Problem solvers use self-regulation strategies to:

- Tell themselves what to do.
- Ask themselves questions.
- Recall what they know.
- Detect and correct errors.
- Monitor their performance.

Self-regulation strategies help problem solvers gain access to processes and strategic knowledge, guide learners as they apply strategies, and regulate their use of strategies and their overall performance as they solve problems. They can be used overtly (talking out loud or whispering to oneself) or covertly (silent self-talk). Metacognitive or self-regulation strategies in the **Solve It!** process include:

- **Say**: Self-instruction implies telling oneself what to do before and while performing actions.
- **Ask**: Self-questioning means asking oneself questions while engaged in an activity in order to stay on task, regulate performance, and verify accuracy.
- **Check**: Self-monitoring requires the problem solver to make certain that everything is done correctly throughout the problem-solving process.

The text box, ***Solve It!*** *Math Cognitive Processes and Self-Regulation Strategies*, shows the cognitive processes and metacognitive strategies used in the **Solve It!** program. It is important to note that as students become familiar with the **Solve It!** process and proficient at solving problems, these math cognitive processes and self-regulation strategies become internalized and automatic.

Solve It! Math Cognitive Processes and Self-Regulation Strategies

Read (for understanding)

Say: Read the problem. If I don't understand, read it again.

Ask: Have I read and understood the problem?

Check: For understanding as I solve the problem.

Paraphrase (your own words)

Say: Underline the important information. Put the problem into my own words.

Ask: Have I underlined the important information? What is the question? What am I looking for?

Check: That the information goes with the question.

Visualize (a picture or a diagram)

Say: Make a drawing or a diagram. Show the relationships among the problem parts.

Ask: Does the picture fit the problem? Did I show the relationships?

Check: The picture against the problem information.

Hypothesize (a plan to solve the problem)

Say: Decide how many steps and operations are needed. Write the operation symbols (+, –, ×, and ÷).

Ask: If I ___, what will I get? If I ___, then what do I need to do next? How many steps are needed?

Check: That the plan makes sense.

Estimate (predict the answer)

Say: Round the numbers, do the problem in my head, and write the estimate.

Ask: Did I round up or down? Did I write the estimate?

Check: That I used the important information.

Compute (do the arithmetic)

Say: Do the operations in the right order.

Ask: How does my answer compare with my estimate? Does my answer make sense? Are the decimals or money signs in the right places?

Check: That all the operations were done in the right order.

Check (make sure everything is right)

Say: Check the plan to make sure it is right. Check the computation.

Ask: Have I checked every step? Have I checked the computation? Is my answer right?

Check: That everything is right. If not, go back. Ask for help if I need it.